A Multilevel Multiscale Mimetic (M^3) Method for Two-phase Flows in Porous Media

Konstantin Lipnikov, David Moulton, Daniil Svyatskiy, T-7

Flow simulations in porous media involve a wide range of strongly coupled scales. The length scale of short and narrow channels is on the order of millimeters, while the size of a simulation domain may be several kilometers (the richest oil reservoir in Saudi Arabia, Ghawar, is 280 km x 30 km). The permeability of rock formations is highly heterogeneous and may span several orders of magnitude, from nearly impermeable barriers to high-permeable flow channels. For such complex systems fully resolved simulations become computationally intractable. To address this problem we developed the new multilevel multiscale mimetic (M^3) method [1]. This method posseses several distinctive features that lead to more reliable, robust, and efficient simulations of subsurface flows:

Upscaled model. Using the same mathematical model with averaged parameters to perform simulations at a much coarser scale does not adequately capture the influence of the fine-scale structure. In contrast, the M^3 method constructs a hierarchical sequence of coarse-scale models, which provides a framework to capture fine-scale effects more accurately.

Multilevel hierarchy. Many different model upscaling approaches have been proposed [2,3]. All of these methods, except the multilevel upscaling (MLUPS) method [3], consider a two-level structure: coarse- and fine-scale grids. Using a two-level structure most multiscale methods achieve a coarsening factor of approximately 10 in each coordinate direction, while the trends in fine-scale realizations of large reservoirs require a coarsening factor of 100 or more. Using the multilevel hierarchy of the M^3 method we achieve large coarsening factors with small computational cost.

Locally conservative velocity field. A multilevel framework was developed in the MLUPS method, but this approach does not produce conservative velocity fields. This is a crucial requirement for modeling two-phase flows, as these are described by a coupled system of equations for pressure (elliptic) and water saturation (hyperbolic). The M^3 method provides locally conservative velocity fields on all scales, which guarantees local mass conservation.

Algebraic nature. We merge two computational strategies that were never used for two-phase flow simulations. The first strategy is the algebraic coarsening developed by Y. Kuznetsov that reduces the degrees of freedom inside a coarse-grid cell [4]. The second is a *novel* approach to conservative coarsening of velocities on the edges of a coarse-grid cell. These complementary strategies ensure that the coarse-scale system has the same sparsity structure as the fine-scale system, which naturally leads to a multilevel algorithm. Due to its algebraic nature, the method can be adapted to other fine-scale discretizations, such as the mixed finite element and finite volume methods and can handle full permeability tensors and general polygonal meshes.

Conservative coarsening. The conservative coarsening procedure is defined by velocity coarsening parameters. These parameters play a critical role in the accuracy of the M^3 method. We implement a black-box, problem-dependent, and computationally inexpensive strategy to estimate them. In most multiscale methods the specific parameters that define the coarsening procedure are computed at the initial time step, with high accuracy, and are not changed during the simulation. Our numerical experiments demonstrate that it is important to update these parameters in time, even with moderate accuracy, Thus we propose to update our velocity coarsening parameters a few times during the simulation (e.g., every 500 time steps) using an efficient algebraic multigrid algorithm with a modest convergence tolerance. With this update strategy the error in the M^3 solution is comparable to the error in the fine-scale solution.

The M^3 method has been applied to the upscaling benchmark from the 10th SPE comparative solution project. We simulated flow in the fluvial layer shown in Fig. 1. (left) with the five-spot well configuration shown in Fig. 2 (right). The permeability field has large channelized structures, which is a challenging problem for multiscale methods. To discretize this system in time we use the IMPES time discretization scheme (implicit pressure and explicit saturation). The saturation is updated using the Darcy velocities provided by the pressure solver.

The numerical results demonstrate that with a large coarsening factor, such as 30, the M^3 solution is close to the fine-scale solution (see Fig. 2). In other numerical tests for larger problems we implement more aggressive coarsening with a factor of 64, and also observe good agreement with the fine-scale solution. The M^3 method speeds up the pressure solver up to 80 times, and the overall simulation eight times, with respect to the fine-scale simulation.

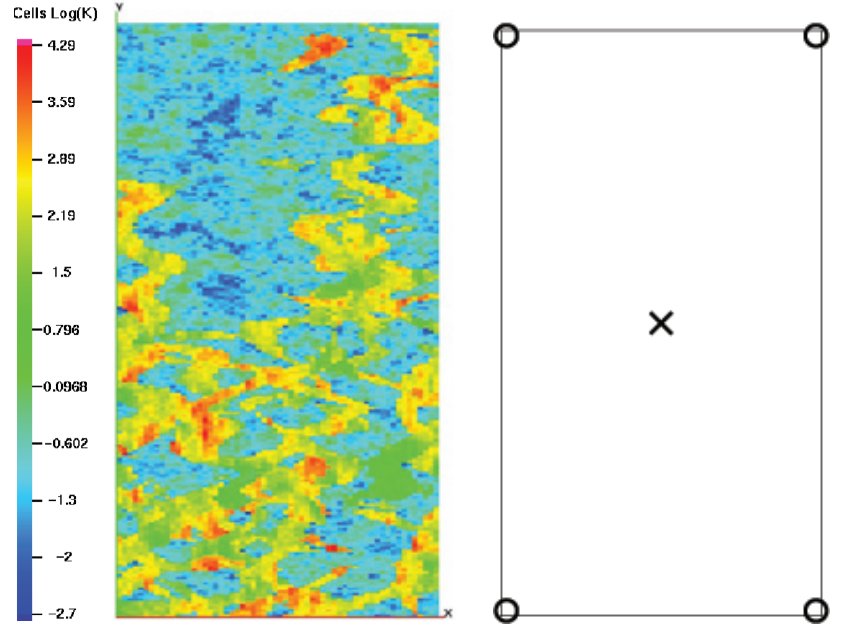

Fig. 1. The absolute permeability, K, of the 68th layer in an SPE benchmark model (left). Locations of the injector (x) and the producers (o) (right).

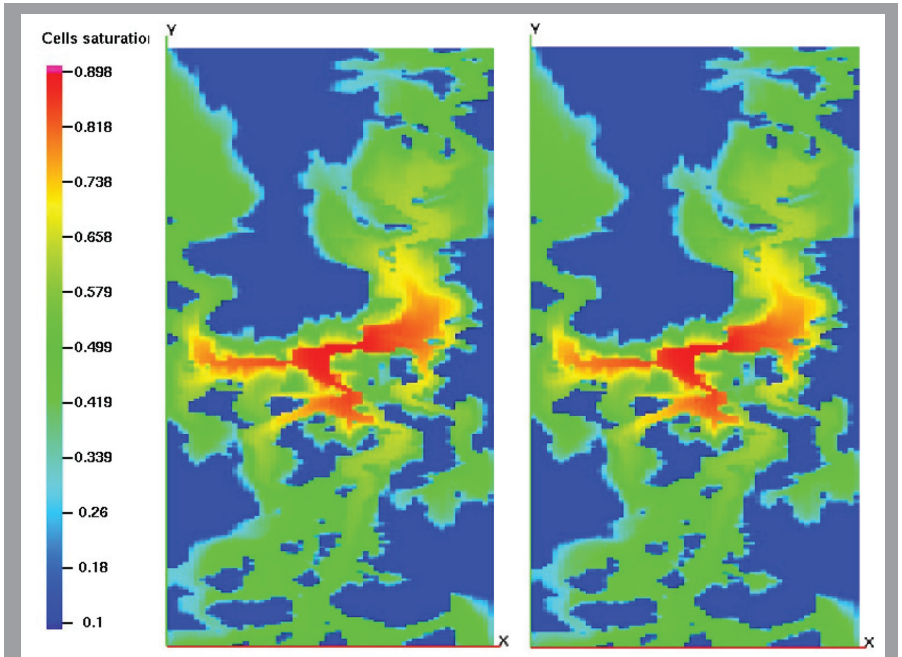

Fig. 2. The water saturation in the two-phase immiscible flow model after 880 days with injection of 200 ft^3 of water per day. The multiscale solution (right) preserves important features of the fine-scale solution (left). The pressure equation was solved on 7 x 2 and 220 x 60 meshes, respectively. The speedup of the pressure solver is 60 times.

For more information contact David Moulton at moulton@lanl.gov.

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